# DRYING OF A MOIST CAPILLARY-POROUS BODY IN MOVING AIR

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#### *(Received 18 May 1965)*

Abstract—The drying of a semi-infinite porous body in contact with the moving fluid has been studied theoretically. The equations of Luikov and Mikhailov for unsteady internal heat and mass transfer in the porous medium together with energy and flow equations of the fluid including the frictional heating have been solved. The fluid has been taken to be incompressible and the motion is assumed to have started impulsively. The effect of Eckert number on the temperature and the mass-transfer potential has been exhibited graphically for a given set of values of various non-dimensional parameters.

#### NOMENCLATURE

- length co-ordinate perpendicular to the  $x,$ direction of flow [m];
- temperature  $[°C]$ ; t.
- θ. moisture-transfer potential [°M];
- air velocity  $[m/s]$ ;  $v,$
- thermal diffusivity coefficient  $[m<sup>2</sup>/h]$ ;  $a<sub>q</sub>$
- diffusion coefficient of moisture in  $a_{m}$ capillary porous body  $[m^2/h]$ ;
- coefficient of moisture internal evapora- $\epsilon,$ tion;
- specific heat of evaporation [kcal/kg]; ρ,
- δ. thermal gradient coefficient [l/degC];
- $= \delta/c_m$  Soret coefficient ["M/degC]; δ,,
- specific isothermal mass capacity of  $c_m$ moist body [kg/kg "Ml;
- specific heat capacity of moist body  $c_{q}$  $[kcal/(kg degC)]$ :
- mass-transfer coefficient  $a_m$  $\left[\frac{\text{kg}}{\text{m}^2} \text{h} \text{°M}\right]$ ;
- dynamic viscosity [kg/(m h)]; μ,
- kinematic viscosity  $[m^2/h]$ ; ν,
- time [h]; Τ,

$$
\lambda_q
$$
, thermal conductivity [kcal/(m h degC)];

- $\lambda_m$  $= a_m c_m \gamma$  moisture conductivity  $[kg/(m h<sup>o</sup>M)]$ ;
- the density of dry porous medium  $\gamma$ ,  $[kg/m<sup>3</sup>]$ :
- L. an arbitrary depth of the porous body [ml;

$$
Lu = a_{m2}/a_{q2}
$$
 Luikov number;

 $Ko = \rho c_{m2}(\theta_{20} - \theta_p)/c_{q2}(t_{10}-t_{20})$  Kossovich number;

$$
Pn = \delta_8(t_{10} - t_{20})/(\theta_{20} - \theta_p)
$$
 Posnov number;

$$
Bi_m = a_m L / \lambda_m
$$
 Biot number for mass trans-  
fer;

- $Pr = v/a_{q1}$  Prandtl number;
- $E = v_0^2/c_{q1}(t_{10} t_{20})$  Eckert number;
- $Fo = a_{q2}\tau/L^2$  Fourier number;
- $V = v/v_0$  dimensionless velocity;
- $T = (t t_{20})/(t_{10} t_{20})$  dimensionless temperature;
- $\theta = (\theta_{20} \theta)/(\theta_{20} \theta_p)$  dimensionless masstransfer potential;
- $\theta_p$ , equilibrium value of mass-transfer potential;
- $X = x/L$  dimensionless length co-ordinate;

$$
k = \lambda_{q1}/\lambda_{q2}
$$

 $K_0(x)$ , modified Bessel function of second kind and of order zero.

# Subscripts

- 1, fluid;<br>2, porou
- 2, porous body;<br>0, initial value.
- initial value.

# 1. INTRODUCTION

LUIKOV and Mikhailov [1] have studied a number of problems of drying of capillary-porous bodies where the set of coupled equations of internal

heat and mass transfer have been solved with various boundary and initial conditions. The most general boundary conditions are those of the third kind which take into account the thermo-diffusion and the heat loss due to evaporation of the liquid moisture at the surface of the body. Lebedev [2] has discussed some empirical formulae for heat and mass transfer in the drying of a moist porous body in air.

In this paper we discuss the problem of drying of a capillary-porous body of infinite extent by means of air flowing on its surface. In all the cases of drying hitherto discussed the surrounding medium is taken to be at a constant temperature or at a temperature given as a function of time. In general, the flowand the energyequations for the moving fluid are coupled through nonlinear convective terms and no analytical solution to the heat transfer in a fluid can be obtained, In one-dimensional flow, however, the convective terms vanish identically and the velocity satisfies a diffusion type of equation. ln this case the flow equation can be solved independently and the solution can be substituted in the energy equation to obtain the temperature time history in the solid and the fluid. In this case the solid and the fluid regions can be treated as a composite medium satisfying the continuity relations at the interface. In the problem discussed here the porous medium is supposed to be dried at its surface by air (considered as incompressible fluid) moving with initial velocity  $v_0$ . At the surface we consider the boundary conditions of continuity of temperature and of heat flux. As for the boundary conditions of mass transfer we consider the interface to be a free surface and assume that the body is in contact with a surrounding medium with a different masstransfer potential which is constant. The equations of internal heat and mass transfer (including the cross effects) have therefore been solved along with the unsteady flow and energy equations of the fluid.

For a particular case, numerical estimates of the heat- and mass-transfer potentials at the surface of the porous body for various values of Eckert numbers and the variation of these potentials with the Fourier number and nondimensional distance inside the body have been graphically exhibited.

### 2. THE PROBLEM

Consider a semi-infinite porous body  $(x < 0)$ initially at a temperature  $t_{20}$  and moisture transfer potential  $\theta_{20}$  in contact with a fluid occupying the space  $x > 0$ . The fluid (air) is initially at a temperature  $t_{10}$ , and it suddenly starts moving with a velocity  $v_0$ . Assuming that moisture transfer at the interface follows the convective transfer law and the continuity conditions of temperature and heat flux hold, determine the temperature and mass-transfer potential in the porous body and the temperature in the moving fluid at any time.

For the solution of the above problem we consider the following system of equations and boundary conditions. Equations of internal heat and mass transfer in the porous medium :

$$
\frac{\partial t_2}{\partial \tau} = a_{q2} \frac{\partial^2 t_2}{\partial x^2} + (\epsilon \rho c_{m2} / c_{q2}) \frac{\partial \theta_2}{\partial \tau}
$$
 (1)

$$
\frac{\partial \theta_2}{\partial \tau} = a_{m2} \frac{\partial^2 \theta_2}{\partial x^2} + a_{m2} \delta_s \frac{\partial^2 t_2}{\partial x^2} \tag{2}
$$

$$
x < 0, \quad \tau > 0
$$

Flow and energy equations of the fluid *:* 

$$
\frac{\partial v}{\partial \tau} = \nu \frac{\partial^2 v}{\partial x^2} \tag{3}
$$

$$
\frac{\partial t_1}{\partial \tau} = a_{q1} \frac{\partial^2 t_1}{\partial x^2} + (\mu/\gamma_1 c_{q1}) \left(\frac{\partial v}{\partial x}\right)^2 \qquad (4)
$$

$$
x > 0, \quad \tau > 0
$$

Initial conditions :

$$
\begin{array}{l}\n t_2 = t_{20} \\
 \theta_2 = \theta_{20}\n \end{array}\n \bigg|\n \begin{array}{l}\n x < 0\n \end{array}\n \tag{5}
$$

$$
\begin{array}{c}\n t_1 = t_{10} \\
 v = v_0\n \end{array}\n \bigg] x > 0
$$
\n(7)\n(8)

Boundary conditions at  $x = 0$ :

$$
t_1 = t_2 \tag{9}
$$

$$
- \lambda_{q2} \frac{\partial t_2}{\partial x} - (1 - \epsilon) \rho a_{m2} (\theta_2 - \theta_p)
$$
  
= 
$$
- \lambda_{q1} \frac{\partial t_1}{\partial x} (10)
$$

$$
\lambda_{m2}\frac{\partial\theta_2}{\partial x} + \lambda_{m2}\delta_s\frac{\partial t_2}{\partial x} + \alpha_{m2}(\theta_2 - \theta_p) = 0 \quad (11)
$$

$$
v = 0 \tag{12}
$$

The boundary condition (11) is the usual massbalance equation,\* while equation (10) is the balance equation, while equation (10) is the Multiplying the above equations (13), (14) and energy-balance equation at the interface where also the boundary conditions (21)–(23) by the heat loss due to evaporation has been taken into account.

The above equations can be represented in the non-dimensional form as below:

$$
\frac{\partial T_2}{\partial F_0} = \frac{\partial^2 T_2}{\partial X^2} - \epsilon K_0 \frac{\partial \theta_2}{\partial F_0} \tag{13}
$$
\n
$$
p\theta_2 = L u \frac{d^2 \theta_2}{d^2 F_0} - L u P n \frac{d^2 \overline{T_2}}{d^2 F_0}
$$

$$
\frac{\partial \theta_2}{\partial F_o} = Lu \frac{\partial^2 \theta_2}{\partial X^2} - LuPn \frac{\partial^2 T_2}{\partial X^2} \qquad (14)
$$

$$
X < 0, \quad Fo > 0 \qquad \frac{\mathrm{d}T}{\mathrm{d}X}
$$

$$
\frac{\partial V}{\partial T} = a \cdot Pr \frac{\partial^2 V}{\partial T} \tag{15}
$$

$$
\frac{\partial T_1}{\partial F_0} = a \frac{\partial^2 T_1}{\partial X^2} + Pr \cdot E \cdot a \left(\frac{\partial V}{\partial X}\right)^2 \quad (16)
$$

Initial conditions:

$$
\begin{array}{c}\nT_2 = 0 \\
\theta_2 = 0\n\end{array}\n\right)\n\quad\nX < 0
$$
\n(17)

\nwhere

\n
$$
\begin{array}{c}\n(17) \\
(18)\n\end{array}\n\quad \text{where}
$$

$$
\begin{array}{c}\nT_1 = 1 \\
V = 1\n\end{array}\n\begin{array}{c}\nX > 0\n\end{array}\n\tag{19}
$$

Boundary conditions at  $X = 0$ :

$$
T_1 = T_2 \tag{21}
$$

$$
\frac{\partial T_2}{\partial X} + (1 - \epsilon) \text{LuKoBin} \left( 1 - \theta_2 \right) = k \frac{\partial T_1}{\partial X} \text{ (22)}
$$

$$
-\frac{\partial \theta_2}{\partial X} + Pn \frac{\partial T_2}{\partial X} + Bi_m (1 \quad \theta_2) = 0 \quad (23)
$$

$$
V = 0 \tag{24}
$$

where the dimensionless temperature, masstransfer potential, etc., have been defined in the

**H.M.-G.** 

# 3. SOLUTION OF THE PROBLEM

Let the Laplace transform of a function  $\varphi(X, Fo)$  be defined as

$$
\bar{\varphi}(X,p) = \int_{0}^{\infty} \varphi(X, Fo) \exp[-pFo] dFo \quad (25)
$$

 $\exp[-pF_0]$  and integrating [with the initial conditions (17), (18)] we get

$$
p\overline{T}_2 = \frac{\mathrm{d}^2 \overline{T}_2}{\mathrm{d} X^2} - \epsilon K \overline{c} p \overline{\theta}_2 \tag{26}
$$

$$
p\theta_2 = Lu\frac{\mathrm{d}^2\theta_2}{\mathrm{d}X^2} - LuPn\frac{\mathrm{d}^2\bar{T}_2}{\mathrm{d}X^2} \tag{27}
$$

$$
(14) \hspace{3.1em} \overline{T}_1 = \overline{T}_2 \hspace{3.1em} (28)
$$

$$
\frac{\mathrm{d}T_2}{\mathrm{d}X} + (1 - \epsilon) \text{ Lukobim} \ (1 - \bar{\theta}_2) = k \frac{\mathrm{d}T_1}{\mathrm{d}X} \quad (29)
$$

$$
\frac{\partial V}{\partial F_0} = a \cdot Pr \frac{\partial^2 V}{\partial X^2} \qquad (15) \qquad -\frac{\mathrm{d}\hat{\theta}_2}{\mathrm{d}X} + P_n \frac{\mathrm{d}T_2}{\mathrm{d}X} + B_i \qquad (1 - \hat{\theta}_2) = 0 \quad (30)
$$

 $\frac{\partial F_1}{\partial F_0} = a \frac{\partial^2 F_1}{\partial X^2} + Pr \cdot E \cdot a \left(\frac{\partial F}{\partial X}\right)^2$  (16) The solution of (15) subject to the initial con-<br>dition (20) and boundary condition (24) is well- $X > 0$ ,  $Fo > 0$  known and is given as

$$
V = \text{erf}\left[\frac{X}{2\sqrt{(Pr \cdot a \cdot Fo)}}\right]
$$
 (31)

**XI2v'U'r. a.** *Fo)* 

where

(19) 
$$
\operatorname{erf}\left[\frac{X}{2\sqrt{(Pr \cdot a \cdot Fo)}}\right] = 2/\sqrt{\pi} \int_{0}^{2\sqrt{(Pr \cdot a \cdot Fo)}} \exp\left[-\frac{e^{\alpha}}{2}\right] d\xi \quad (32)
$$

Substituting  $V$  from (31) into equation (16) and  $T_1 = T_2$  (21) taking the Laplace transform we get

$$
\frac{d^2T_1}{dX^2} - p/a \, T_1
$$
\n
$$
= -\frac{1}{a} - \frac{2E}{\pi a K_0} \left[ \sqrt{\left(\frac{2p}{Pr \cdot a}\right) X} \right] \tag{33}
$$

The solution of simultaneous equations (26) and (27) is

where the dimensionless temperature, mass-  
\ntransfer potential, etc., have been defined in the  
\nmomental, etc., have been defined in the  
\nmomental, etc., have been defined in the  
\n
$$
\delta_2 = -1/\epsilon K \sigma \{A_1 (1 - \beta_1^2) \exp [\beta_1 X \sqrt{p}] + A_2 (1 - \beta_2^2) \exp [\beta_2 X \sqrt{p}] \}
$$
\n\*See reference 1, p. 498.  
\n
$$
X < 0
$$
 (35)

<sup>\*</sup> See **reference 1,** p. **498. x < 0** *(3.5)* 

and of equation (33) is

$$
\tilde{T}_1 = B_1 \exp\left[-qX\right] - E/ \pi a q \exp\left[qX\right]
$$
\n
$$
\int_{-\infty}^{\pi} K_0 \left[\sqrt{\frac{2}{P}}r\right] q \xi \exp\left[-q\xi\right] d\xi
$$
\n
$$
+ E/ \pi a q \exp\left[-qX\right] \int_{0}^{\pi} K_0 \left[\sqrt{\frac{2}{P}}r\right] q \xi
$$
\n
$$
\exp\left[q\xi\right] d\xi
$$
\n
$$
X > 0
$$
\n(36)

where

$$
\beta_1 = (1/2^{\frac{1}{2}}) \{ (1 + 1/Lu + \epsilon KoPn) + \sqrt{[(1 + 1/Lu + \epsilon KoPn)^2 - 4/Lu]} \}^{\frac{1}{2}}
$$

$$
\beta_2 = (1/2^{\frac{1}{2}}) \{ (1 + 1/Lu + \epsilon KoPn) - \sqrt{[(1 + 1/Lu + \epsilon KoPn)^2 - 4/Lu]} \}^{\frac{1}{2}}
$$

and

 $q = \sqrt{p/a}$ 

Satisfying the transformed boundary conditions (28)-(30) by (34)-(36) we get

$$
\begin{aligned} \bar{T}_2 &= \left[ \frac{N_{11}}{p(q+h)} + \frac{N_{12}}{q(q+h)} \right] \exp\left[\beta_1 \, X \sqrt{p}\right] \\ &+ \left[ \frac{N_{21}}{p(q+h)} - \frac{N_{22}}{q(q+h)} \right] \exp\left[\beta \, X_2 \sqrt{p}\right] \\ &X < 0 \quad (37) \end{aligned}
$$

$$
\theta_2 = (-1/\epsilon Ko) \left\{ \left[ \frac{N_{11}}{p(q+h)} + \frac{N_{12}}{q(q+h)} \right] \right\}
$$
  
(1 - \beta\_1^2) exp [\beta\_1 X\sqrt{p}]  

$$
\left[ \frac{N_{21}}{p(q+h)} - \frac{N_{22}}{q(q+h)} \right]
$$
  
(1 - \beta\_2^2) exp [\beta\_2 X\sqrt{p}]  

$$
X < 0
$$
 (38)

and

$$
T_{1} = \left[\frac{N_{31}}{p(q+h)} + \frac{N_{32}}{q(q+h)}\right] \exp\left[-qX\right] + \left[\frac{F}{mqq + h}\right] \exp\left[-qX\right] + \left[\frac{F}{mqq} \exp\left[qX\right] \int_{-\infty}^{x} Ko\left[\sqrt{\frac{2}{r}r}q\xi\right] + \left[\frac{F}{mqq} \exp\left[-qX\right] \right] + \left[\frac{F}{mdq} \exp\left[\frac{F}{dq}\right] \exp\left[q\xi\right] \right] + \left[\frac{F}{mdq} \exp\left[q\xi\right] \exp\left[q\xi\right] + \left[\frac{F}{mdq} \exp\left[q\xi\right] \right] \right] + \left[\frac{F}{mdq} \exp\left[-qX\right] \left[\frac{F}{mdq}\right] + \left[\frac{F}{mdq} \exp\left[-qX\right] \right] + \left[\frac{F}{mdq} \exp\left[-qX\right] \right] + \left[\frac{F}{mdq} \exp\left[-qX\right] \left[\frac{F}{mdq}\right] + \left[\frac{F}{mdq} \exp\left[-qX\right] \right] \right] + \left[\frac{F}{mdq} \exp\left[-qX\right] \left[\frac{F}{mdq}\right] + \left[\frac{F}{mdq} \exp\left[-qX\right] \right] + \left[\frac{F}{mdq} \exp\left[-qX\right] \right] + \left[\frac{F}{mdq} \exp\left[-qX\right] \left[\frac{F}{mdq}\right] + \left[\frac{F}{mdq} \exp\left[-qX\right] \right] \right] + \left[\frac{F}{mdq} \exp\left[-qX\right] \left[\frac{F}{mdq} \right] + \left[\frac{F}{mdq} \exp\left[-qX\right] \left[\frac{F}{mdq}\right] + \left[\frac{F}{mdq} \exp\left[-qX\right] \right] \right] + \left[\frac{F}{mdq} \exp\left[-qX\right] \left[\frac{F}{mdq} \right] + \left[\frac{F}{mdq} \exp\left[-qX\right] \left[\frac{F}{mdq}\right] + \left[\frac{F}{mdq} \right] \right] \right]
$$

where

$$
\sqrt{a} N_{11} S_1 = \beta_2 B i_m - (1 - \epsilon) L u K \omega B i_m
$$

$$
\left[ \frac{\beta_2 (1 - \beta_2^2)}{\epsilon K \omega} + \beta_2 P n \right] + \frac{k}{\sqrt{a}}
$$

$$
(1 + 2EM/\pi) \frac{B i_m}{\epsilon K \omega} (1 - \beta_2^2) + \frac{k}{\sqrt{a}} B i_m.
$$
 (40)

$$
a \cdot N_{12} \cdot S_1 = \frac{\kappa}{\sqrt{a}} (1 + 2EM/\pi) \left[ \frac{\beta_2 (1 - \beta_2^2)}{\epsilon K \theta} + \beta_2 P n \right] \tag{41}
$$

$$
\sqrt{a} N_{21} S_1 = (1 - \epsilon) L u K o B i_m
$$
\n
$$
\left[ \frac{\beta_1 (1 - \beta_1^2)}{\epsilon K o} + \beta_1 P n \right]
$$
\n
$$
- \frac{k}{\sqrt{a}} (1 + 2 E M / \pi) \frac{B i_m}{\epsilon K o} (1 - \beta_1^2)
$$
\n
$$
- B i_m \beta_1 - \frac{k}{\sqrt{a}} B i_m
$$
\n(42)

$$
a \cdot N_{22}S_1 = \frac{\kappa}{\sqrt{a}}(1 + 2EM/n)
$$

$$
\left[\frac{\beta_1(1 - \beta_1^2)}{\epsilon K o} + \beta_1 P n\right] \quad (43)
$$

$$
\sqrt{a} N_{31} S_1 = \frac{k}{\sqrt{a}} \frac{Bi_m}{\epsilon K \sigma} (EM/\pi) (\beta_1^2 - \beta_2^2)
$$
  
\n
$$
- (1 - \epsilon) L u K \sigma Bi_m
$$
  
\n
$$
\left[ \frac{\beta_2 (1 - \beta_2^2) - \beta_1 (1 - \beta_1^2)}{\epsilon K \sigma} + \frac{\beta i_m}{\epsilon K \sigma} (\beta_2 - \beta_1) + Bi_m (\beta_2 - \beta_1)
$$
  
\n
$$
+ \frac{Bi_m}{\epsilon K \sigma} (1 + EM/\pi) [\beta_2 (1 - \beta_1^2)]
$$
  
\n
$$
- \beta_1 (1 - \beta_2^2)] + \frac{(1 - \epsilon)}{\epsilon} L u Bi_m
$$
  
\n
$$
(1 + EM/\pi) \left\{ \frac{(1 - \beta_1^2)(1 - \beta_2^2)(\beta_1 - \beta_2)}{\epsilon K \sigma} + P_n [\beta_1 (1 - \beta_2^2) - \beta_2 (1 - \beta_1^2)] \right\}
$$
  
\n(1 +  $EM/\pi$ )

$$
a \cdot N_{32}S_1 = \frac{k}{\sqrt{a}} (EM/\pi)
$$
  
\n
$$
\left[ \frac{\beta_2(1-\beta_2^2) - \beta_1(1-\beta_1^2)}{\epsilon K_0} + (\beta_2 - \beta_1) Pn + (1 + EM/\pi) + (\beta_1\beta_2/\epsilon K_0) (\beta_2^2 - \beta_1^2)
$$
\n(45)

and

$$
h = \frac{S_2}{(\sqrt{a}) \cdot S_1} \tag{46}
$$

where

$$
S_1 = \frac{k}{\sqrt{a}} \left[ \frac{\beta_2 (1 - \beta_2^2) - \beta_1 (1 - \beta_1^2)}{\epsilon K \sigma} + (\beta_2 - \beta_1) P n \right] + (\beta_1 \beta_2 / \epsilon K \sigma) (\beta_1^2 - \beta_2^2) \tag{47}
$$

$$
S_2 = \frac{Bim}{\epsilon K \sigma} \left[ \frac{k}{\sqrt{a}} (\beta_1^2 - \beta_2^2) + \beta_1 (1 - \beta_2^2) - \beta_2 (1 - \beta_1^2) \right] + \frac{(1 - \epsilon)}{\epsilon} L u Bim
$$
  

$$
\left\{ \frac{(1 - \beta_1^2) (1 - \beta_2^2) (\beta_2 - \beta_1)}{\epsilon K \sigma} + P_n [\beta_2 (1 - \beta_1^2) - \beta_1 (1 - \beta_2^2)] \right\}
$$
(48)

 $M$  has two different values [3] according as the Prandtl number is less than or greater than 2, namely

$$
M_1 = \frac{1}{\sqrt{(1 - 2/Pr)}} \log \left[ \frac{1 + \sqrt{(1 - 2/Pr)}}{\sqrt{(2/Pr)}} \right]
$$
  
Pr > 2 (49)  

$$
M_2 = \frac{1}{\sqrt{(1 - 2/Pr)}} \cos^{-1} [1/(Pr/2)]
$$

$$
M_2 = \frac{1}{\sqrt{(2/Pr - 1)}} \cos^{-1} [\sqrt{(Pr/2)}]
$$
  
Pr < 2 (50)

Applying the Laplace inversion theorem [4] to  $(37)$ – $(39)$ 

$$
T_2(X, Fo) =
$$
\n
$$
\frac{N_{11}}{h} \left\{ \text{erfc} \left( \frac{-\beta_1 X}{2Fo} \right) \exp \left[ -\beta_1 (\sqrt{a}) hX \right] + aFoh^2 \right\} \times \text{erfc} \left[ -\frac{\beta_1 X}{2\sqrt{Fo}} + h\sqrt{(aFo)} \right] \right\}
$$
\n
$$
+ aN_{12} \exp \left[ -\beta_1 (\sqrt{a}) h \cdot X + aFoh^2 \right]
$$
\n
$$
\times \text{erfc} \left[ -\frac{\beta_1 X}{2\sqrt{Fo}} + h\sqrt{(aFo)} + \frac{N_{21}}{h} \right]
$$
\n
$$
\left\{ \text{erfc} \left( \frac{-\beta_2 X}{2\sqrt{Fo}} \right) - \exp \left[ -\beta_2 (\sqrt{a}) h \cdot X \right] + aFoh^2 \right\} \times \text{erfc} \left[ -\frac{\beta_2 \cdot X}{2\sqrt{Fo}} + h\sqrt{(aFo)} \right] \right\}
$$
\n
$$
- N_{22} \cdot a \exp \left[ -\beta_2 (\sqrt{a}) h \cdot X + aFoh^2 \right]
$$
\n
$$
\times \text{erfc} \left[ -\frac{\beta_2 X}{2\sqrt{Fo}} + h\sqrt{(aFo)} \right]
$$
\n
$$
X < 0
$$

$$
\theta_2(X, Fo) = -\frac{N_{11}}{h} \frac{(1 - \beta_1^2)}{\epsilon K0}
$$
\n
$$
\left\{ \text{erfc} \left( \frac{-\beta_1 X}{2\sqrt{Fo}} \right) - \text{exp} \left[ -\beta_1 (\sqrt{a}) h \cdot X \right] \right\}
$$
\n
$$
+ aFoh^2 \right] \times \text{erfc} \left[ \frac{-\beta_1 X}{2\sqrt{Fo}} + h\sqrt{(aFo)} \right] \}
$$
\n
$$
-\frac{a \cdot N_{12} (1 - \beta_1^2)}{\epsilon K0} \exp \left[ -\beta_1 (\sqrt{a}) h \cdot X \right]
$$
\n
$$
+ aFoh^2 \right] \times \text{erfc} \left[ \frac{-\beta_1 X}{2\sqrt{Fo}} + h\sqrt{(aFo)} \right]
$$
\n
$$
-\frac{N_{21} (1 - \beta_2^2)}{h} \left\{ \text{erfc} \left( \frac{-\beta_2 X}{2\sqrt{Fo}} \right) \right\}
$$
\n
$$
- \exp \left[ -\beta_2 (\sqrt{a}) h \cdot X + aFoh^2 \right]
$$
\n
$$
\times \text{erfc} \left[ \frac{-\beta_2 \cdot X}{2\sqrt{Fo}} \sqrt{(aFo)} \right] \right\}
$$
\n
$$
+ \frac{a \cdot N_{22}}{\epsilon K0} (1 - \beta_2^2) \exp \left[ -\beta_2 (\sqrt{a}) h \cdot X + \frac{a Foh^2}{2\sqrt{Fo}} \right].
$$
\n
$$
aFoh^2 \right] \times \text{erfc} \left[ \frac{-\beta_2 X}{2\sqrt{Fo}} + h\sqrt{(aFo)} \right].
$$

$$
T_1(X, Fo) = 1 + \frac{N_{31}}{h} \left\{ \text{erfc} \left[ \frac{X}{2\sqrt{(aFo)}} \right] - \exp [h \cdot X + aFoh^2] \times \text{erfc} \left[ \frac{X}{2\sqrt{(aFo)}} \right] + h\sqrt{(aFo)} \right\} + a \cdot N_{32} \exp [h \cdot X + aFoh^2] \cdot \text{erfc} \left[ \frac{X}{2\sqrt{(aFo)}} + h\sqrt{(aFo)} \right] + \frac{EPr^{\frac{2}{3}}}{2\pi} \int_{0}^{F_0} \exp \left\{ -\frac{X^2}{2a[Pr(Fo - \xi) + 2\xi]} \right\} + \frac{\exp \left\{ -\frac{X^2}{2a[Pr(Fo - \xi) + 2\xi]} \right\}}{\sqrt{\{(Fo - \xi)[Pr(Fo - \xi) + 2\xi] \}}} \right\}
$$
\n
$$
\text{erfc} \left( \frac{-\sqrt{[Pr(Fo - \xi)] \cdot X}}{2\sqrt{[a\xi[Pr(Fo - \xi) + 2\xi]})} \right) d\xi
$$
\n
$$
X > 0
$$

where the convolution theorem [5] has been applied to invert the last two terms of (39).

# 4. NUMERICAL RESULTS AND DISCUSSION

From the exact solution to the equations of internal heat and mass transfer in a porous body together with the flow and energy equations of the moisture removing fluid, taking into account the frictional heating, we can write down the temperature and moisture potentials in terms of the various non-dimensional parameters

$$
T_2 = T_2(Lu, Ko, Bi_m, Pn, \epsilon, Fo, a, k, Pr, E)
$$

$$
\theta_2 = \theta_2(Lu, Ko, Bi_m, Pn, \epsilon, Fo, a, k, Pr, E)
$$

For illustration we choose the foIlowing set of values for the non-dimensional parameters

$$
Lu = 0.2
$$
  $Ko = 1.2$   $Ph = 0.5$   $Pr = 0.7$   
 $Bin = 0.1$   $a = 40$   $k = 0.1$ .

The corresponding potentials  $T_2$  and  $\theta_2$  are shown graphically in Figs. l-4.

Figure 1 exhibits the values of the nondimensional moisture transfer potential  $\theta_2$ plotted against the non-dimensional porous body depth for various values of the Fourier



FIG. 1. Variation of  $\theta_2$  along porous body depth for various values of *Fo.*  $(E = 1, \epsilon = 0.5, K_0 = 1.2, Lu =$ 0.2, *Pn =* 0.5, *Pr =* O-7, *k = 0.1, a =* 40, *Bim =* 0.1).



FIG. 2. Effect of Eckert number on the value  $\theta_2$  at the porous surface.



FIG. 3, Variation of *Ta* along porous body depth for various values of *Fo.* 



FIG. 4. Effect of Eckert number on  $T_2$  at the porous body surface.

number. The effect of the Eckert number  $E = v_0^2/c_{q1}(t_{10} - t_{20})$  on the drying process is exhibited in Fig. 2 where the difference in the values of  $\theta_2$  corresponding to a given Eckert number and that corresponding to zero Eckert number is plotted against Fo. Figures 3 and 4 depict the values of non-dimensional temperature in the porous body for variation in X and *Fo* and the effect of the Eckert number on the temperature of the porous surface respectively. It is seen from Fig. 2 that an increase in the Eckert number results in quicker drying though the steady state values of the moisture potential at the surface is independent of the Eckert number. The effect of frictional heating on the surface temperature seems to be more pronounced and is reflected even in the steady state values of temperature at the porous surface. It is observed (Fig. 3) that for the values of non-dimensional parameters chosen for the numerical example illustrated here the temperature of the porous surface rises only slightly above the initial temperature in the beginning but later the cooling due to moisture evaporation from the surface is dominant and the temperature falls down to a steady state value which is dependent on the Eckert number. The rise of temperature of the porous body surface above the initial value is greater for larger Eckert numbers.

#### ACRNOWLEDGEMENTS

The authors are thankful to Dr. R. R. Aggarwal, Asstt. Director, Applied Maths. Division, for his keen interest in the work and to the Director, Defence Science Laboratory for permission to publish this paper.

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Résumé—Le séchage d'un milieu poreux semi-infini en contact avec un fluide en mouvement a été étudié théoriquement. Les équations de Luikov et Mikhailov pour le transport de chaleur et de masse transistoire à l'intérieur du milieu poreux ont été résolues en mêmet emps que les équations de l'écoulement et de l'énergie du fluide en tenant compte de l'échauffement dû au frottement. Le fluide a été supposé incompressible et l'on a supposé que le mouvement a démarré brutalement. L'effet du nombre d'Eckert sur la température et sur le potentiel de transport de masse a été présenté graphiquement pour un ensemble donné de valeurs de différents paramètres sans dimensions.

Zusammenfassung-Die Trocknung eines halbunendlichen porösen Körpers der sich in Berührung mit einem bewegten Medium befindet wurde theoretisch untersucht. Die Gleichungen von Luikov und Mikhaliov für instationären inneren Wärme- und Stoffübergang im porösen Körper wurde zusammen mit den Energie- und Bewegungsgleichungen, die Reibungserwärmung einschliessen, gelöst. Das strömende Medium ist als inkompressibel angenommen und die Bewegung soll plötzlich begonnen haben. Der Einfluss der Eckertzahl auf die Temperatur und das Stoffübergangspotential ist für mehrere dimensionslose Parameter grafisch dargestellt.

Аннотация-Проведено теоретическое исследование сушки полубесконечного пористого тела, находящегося в контакте с движущейся средой. Решены уравнения Лыкова и Михайлова для процесса нестационарного внутреннего тепло-и массопереноса в пористой среде совместно с уравнения энергии и движения жидкости, включающими в себя нагрев от трения. При анализе было принято, что жидкость несжимаема и что движение начинается импульсивно. Для данного ряда значений различных безразмерных параметров представлена графическая зависимость температуры и потенциала массопереноса от критерия Эккерта.